

**William Paterson University
College of Education**

Name: Lisa Warner Topic: Area and Perimeter	School: #5 Essential Question: How are area and perimeter used in real life problems?	Grade: 4
Concept (to be developed and explored by students): Area and Perimeter of rectangles		
Standard/s): Common Core State Standard/s) (content standards): <u>CCSS.MATH.CONTENT.3.MD.C.6:</u> Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). <u>CCSS.MATH.CONTENT.3.MD.C.7:</u> Relate area to the operations of multiplication and addition. <u>CCSS.MATH.CONTENT.3.MD.C.7.A:</u> Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. <u>CCSS.MATH.CONTENT.3.MD.C.7.B:</u> Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. <u>CCSS.MATH.CONTENT.3.MD.D.8:</u> Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. <u>CCSS.MATH.CONTENT.4.MD.A.3</u> Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i> CCSS Mathematical Standards for Practice: The teacher encourages students to: MSP1. Make sense of problems and persevere in solving them. MSP 2. Reason abstractly and quantitatively. MSP3. Construct viable arguments and critique the reasoning of others. MSP4. Model with mathematics. MSP5. Use appropriate tools strategically. MSP6. Attend to precision. MSP7. Look for and make use of structure. MSP8. Look for and express regularity in repeated reasoning. NCTM Mathematics Teaching Practice Standards: Effective mathematics teachers: 1. Establish mathematics goals to focus learning 2. Implement tasks that promote reasoning and problem solving. 3. Use and connect mathematical representations. 4. Facilitate meaningful mathematical discourse. 5. Pose purposeful questions. 6. Build procedural fluency from conceptual understanding. 7. Support productive struggle in learning mathematics. 8. Elicit and use evidence of student thinking.	Learning objective(s): To investigate the meaning of area and perimeter of rectangular polygons including when, how and why to use each, and the relationship between area and perimeter.	Assessment(s): To better understand what students remember about area and perimeter, how they remember it and how they apply those ideas to a context. Data is collected during group work, student presentations, written work and answers to questions posed. Performance task assesses concepts, procedures, reasoning, problem solving, mathematical processes, and communication: Ms. Brown's class will raise rabbits for their spring science fair. They have 12 feet of fencing with which to build a rectangular rabbit pen to keep the rabbits. - Show at least 3 rectangular pens you can make using the 12 feet of fencing (for each pen). - If Ms. Brown's students want their rabbits to have as much room as possible, what is the length and width of the rectangular rabbit pen with the most amount of space inside (using the 12 feet of fencing)? Convince us. - How do area and perimeter relate to this problem? - What method can you use to find the area of each rectangular pen that you created? Why does it work? How do you know it will always work? - What method can you use to find the perimeter of each rectangular pen that you created? Why does it work? How do you know it will always work? Rubric attached.

Planning:

- Materials: colored tiles (plastic or wood), graph paper (with square centimeters), chart paper, colored markers, rulers. Other possibilities that allow for easy measurement may also be considered.
- Technology / Visuals: calculators, document camera (if a document camera is not available, an overhead projector or large chart paper may be used instead).
- Groupings: The students will work individually during the Engage stage and in groups of 2-3 (depending on the total number of students present in class that day) during the Explore Stage. During the Explain and Extend stages, there will be a whole class discussion, with the student presenters in the front of the room most of the time.

- Academic Vocabulary:

There are precise mathematical definitions and there are also mathematically acceptable definitions that children in 4th grade may develop. Both are listed:

Area: the number of squares or square units that cover a two-dimensional shape or closed figure or region or polygon (in this case the number of square tiles or squares on the graph paper); the measurement of the area inside of a two-dimensional shape or closed figure or region or polygon.

Perimeter: the distance around a region or two-dimensional shape or closed figure or polygon (in this case the length of 1 unit is the side of a square tile or square on the graph paper)

Rectangle: a four-sided shape or polygon or closed figure (or quadrilateral) that is made up of two pairs of parallel sides and has four squared corners or right angles or 90 degree angles; a parallelogram with a squared corner or right angle or ninety degree angle

Additional words that may arise in the lesson:

Square: a four sided shape or polygon or closed figure (or quadrilateral) with all right angles or squared corners or ninety degree angles and sides of equal length or congruent sides; a parallelogram that has all right angles or squared corners or ninety degree angles and all sides congruent or equal; a rectangle that has all four sides of equal length or congruent; a four sided shape or polygon or closed figure that has two pairs of parallel sides, four squared corners or right angles or 90 degree angles and all sides equal or congruent

Dimension: is a measure in one direction, such as the length, width, or height of a figure. For example, length and width are the dimensions of a rectangle.

*Some of these definitions are adapted from Van de Walle, et al (2013). *Elementary and middle school mathematics: Teaching developmentally (8th Edition)*.

- Cite Sources:

Chapin, S.H., O'Connor, C. & Anderson, N.C. (2009). *Classroom discussions: Using math talk to help students learn*. Sausalito, CA: Math Solutions Publications.

National Council of Teachers of Mathematics (NCTM) (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

National Governors Association Center for Best Practices (NGA Center) & Council of Chief State School Officers (CCSSO). (2010). *Common Core State Standards*. Washington D.C.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards based mathematics instruction*. New York: Teachers College Press.

Van de Walle, J., Karp, K.S., & Bay-Williams, J.M. (2013). *Elementary and middle school mathematics: Teaching developmentally (8th Edition)*. New York, NY: Addison Wesley Longman.

Teaching and Learning		
Process Steps:	Teacher	Students
Engage (10-15 minutes)	According to the Common Core State Standards, students were introduced to area and perimeter in the 3rd grade (see 3rd grade content standards above). This lesson can be used as a reintroduction to area and perimeter. The teacher will ask the following questions to elicit prior knowledge and provide a context for the activity/task, without lowering the cognitive demand^{1,2}: Have any of you ever seen a rabbit pen before? Would you tell us about it? What did it look like? What else did you notice about it? (The teacher chooses several students to explain and then asks the class what they know about each shape.) The teacher will distribute the word problem and choose a student to read the word problem out loud: Ms. Brown's class will raise rabbits for their spring science fair. They have 12 feet of fencing with which to build a rectangular rabbit pen to keep the rabbits. Show as many rectangular pens as you can make using 12 feet of fencing (for each pen). If Ms. Brown's students want their rabbits to have as much room as possible, what is the the length and width of the rectangular rabbit pen with the most amount of space inside (using the 12 feet of fencing)? Convince us. *Problem adapted from <i>Implementing standards based mathematics instruction</i> (Stein, et al, 2009) The teacher will ask the students questions to help them begin to think about how to approach the problem without lowering the cognitive demand¹: What would an example of a rectangular pen be? The teacher will ask one student to come up to the document camera to create a blueprint of a pen with the colored tiles. The teacher asks the students in the audience: What do you think? Would someone convince us that this rectangle does or doesn't work? How would you draw this rectangle on graph paper? Would someone come to the document camera to show us how and explain your process? How would you label the length? Why? How would you label the width? Why? This is also called the dimensions of your rectangle. Do you think that there are more pens that can be made? Why or why not. What do you think having "as much room as possible" means? What is inside these rectangles? In this blueprint we will call these square feet. The teacher will then establish expectations¹ by telling the students: You will now work in groups of 3 for at least 15 minutes. As you do this, I will listen to your ideas and may ask you questions about your thinking. Each group will then be asked to record their solution on chart paper and present these ideas to the class.	Students may raise their hand from their seats and share examples of pens they saw in real life, and explain what they looked like. All students will have an opportunity to read the problem, then one student will read the problem out loud as the students are sitting at their desks reading the problem silently. Students may raise their hand from their seats and describe a possible rectangle. (For example, one rectangular pen may be a 6ft by 2 ft pen). One student will go up to the document camera to show how to create a rectangular pen with colored tiles. Another student will draw the rectangle onto graph paper. Students in the audience may agree or disagree with these and explain why. The students at the document camera may defend their thinking or explain why they agree with the student in the audience. Other students may explain that there are squares inside these rectangles.

¹ Van de Walle, J., Karp, K.S., & Bay-Williams, J.M. (2013). *Elementary and middle school mathematics: Teaching developmentally (8th Edition)*.

Explore (20-25 minutes)	While the students are working in groups of three developing their own ways of thinking, the teacher is listening to students' ideas, monitoring and assessing progress, providing support, and offering meaningful extensions^{1,2}. Some possible teacher questions: Tell me about your idea. How are you using the colored tiles or graph paper? What does each side of the square represent for you? What does each square in your drawing represent? What does this part of your picture or number sentence mean? I noticed you counted squares, how else might you solve? How does counting squares relate to your number sentence? Why did you multiply? What do you think about your group member's idea? Tell him/her if you agree or disagree and why. How do you know you found all rectangles? How can you be sure there isn't a pen with more space? Why is that the pen with the most space? What would happen if you had 16 feet of fencing to use instead of 12 feet of fencing (for students who finish early)? If the teacher notices that several groups of students are using 12 squares (as the area) for each rectangle instead of 12 feet as the perimeter, the teacher may stop the students while they are working to ask questions about this. For example, the teacher may ask: How are you using the 12 feet? Why do you think it should be used this way? If a student explains that the 12 feet is the perimeter in each rectangle, the teacher will ask, How do you know? The teacher will ask the other students: Do you agree or disagree? Why? If another student says the 12 represents the number of squares inside or area, the teacher will ask: How do you know (even though this is incorrect, the teacher will remain neutral in his/her voice, tone and body language)? (This is a good opportunity for the students to discuss area and perimeter and how to apply each.) If the students do not bring in the word area, the teacher will ask them for a word that measures the space inside or square units inside the rectangle. If they do not know, the teacher will give them the word area for what they are describing. If the students do not bring in the word perimeter when they are describing the distance around the rectangle (in this case, always 12 feet of fencing), the teacher will ask them for a word that measures what they are describing. If they do not know, the teacher will give them the word perimeter. If students finish early, the teacher may ask them to find the rectangular rabbit pen with the most space if they had 16 feet of fencing to use (and then 24 feet of fencing). The teacher asks the groups to organize and write up their solutions on chart paper to prepare for their presentations.	Some students use colored tiles to develop blueprints of rectangular pens. Some students draw their rectangles on graph paper. Some students discover that the perimeter will always be 12 feet in their blueprints. Some students discover that counting the squares inside is a way to find out how much space is inside and they can also use repeated addition or multiplication (columns times the rows or length times width). Some students may work from their previous rectangle by adding 1 to the length and subtracting 1 from the width to get the next rectangle with the same perimeter. Some students may create lists and/or charts that include the length, width and area as they develop strategies for finding all possible rectangular pens using whole numbers. Students show and explain their ideas to each other and the teacher, question their own and others' ideas, defend or justify their ideas and build ideas from each other. Some students may share ideas with the entire class from their seats when/if the teacher stops them to have a conversation about how they are using the 12 feet (for the perimeter or area). Some students may critique the reasoning of others and construct viable arguments. Students work together to organize and write up their solutions on an overhead transparency or chart paper.
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² Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards based mathematics instruction*.

Explain (20-25 minutes)	The teacher asks students to present their ideas in the front of the room. The teacher organizes the sequence of the presentations by idea. For example, the teacher will not ask a group that has a chart (without the drawings) to present first (since the list of numbers is more abstract than the drawings). While students are presenting their solutions in the front of the room, the teacher will promote a community of learners, actively listen to student ideas, use talk moves to help students clarify and share their own thoughts, deepen their reasoning and engage with others' reasoning (e.g. ask the students if they agree or disagree with another student's idea and to explain why)^{1,3}. Some possible teacher questions to students in the audience: Does anyone have a question about this group's thinking? Would anyone like to explain what they are saying in your own words? Are you convinced by their idea? Why or why not? Some possible teacher questions to the presenters: Please tell us more; I am confused by that part of your solution. Would you explain further? Are you saying that the finding the area helped you determine which pen had the most space inside?	Groups of students will present their ideas to the class in the front of the room. After they show and explain their strategies and solutions, they will answer questions from their peers in the audience and teacher, defend their solutions and build on their thinking. Some students may discover that there are several strategies for finding out how much space is inside (eg. counting squares, repeated addition, multiplication) and this is called area. Some students may discover that there are several strategies for confirming that they used exactly 12 feet of fencing outside (eg. counting the sides of their rectangles, adding the length and width and then multiplying by 2; length times 2 plus width times 2) and this is called perimeter.
Extend (10-20 minutes)	The teacher will encourage the students to go deeper into the content and possibly offer additional perimeter lengths to investigate (i.e. 24, etc.) – The teacher will encourage informal generalization and justification, encourage students to connect their ideas to each other, encourage students to summarize main ideas and identify future directions¹. For example, the teacher might ask them if it is possible to have a constant area with the perimeter changing and to create a word problem that would call for that. Some possible teacher questions to students in the audience: How does this compare to your idea, Frank? Is there anyone who would like to go to the front to try to prove that idea wrong? How do their drawings relate to their number sentences? Some possible teacher questions to the presenters: How can you be sure that length times width would work every time? How can you be sure that there isn't a rectangular pen that has more space inside? How can you convince us that you found the rectangular rabbit pen with the most space and that there isn't one that has more space? Why do you think that the square pen is a rectangle? Convince us.	Some ideas that may emerge: Students may explain that even though the perimeter stays the same, the area can change. Students may explain that the square has the most space inside because as you stretch out the shape, you lose space. Students may explain that if they continue their list or chart by increasing the length by one and decreasing the width by one, the rectangles will repeat. Students may explain that multiplying length times width is just a quicker way to count the squares inside to find the area. Students may explain that adding the length and the width and then multiplying by 2 is the same as counting the length of each side to find the area because the lengths of the top and bottom sides of the rectangle are equal to each other and the lengths of the right and left sides of the rectangle are equal each other.

³ Chapin, S.H., O'Connor, C. & Anderson, N.C. (2009). *Classroom discussions: Using math talk to help students learn*. Sausalito, CA: Math Solutions Publications.

		Students may realize that a square is a type of rectangle. All rectangles have 2 pairs of parallel sides and four right angles or squared corners. Students may show that a square has 2 pairs of parallel sides, and four right angles or squared corners. Students may explain that a square is a type of rectangle with all sides that are equal in length or congruent.
Differentiated Instruction: For students with special needs²: If a student has trouble forming a mental representation of the mathematical concepts (eg. has difficulty interpreting the word problem), the teacher may want to consider drawing a rectangle for them to get them started or giving them larger graph paper or asking them more questions about their drawings and how they relate to the problem to help them make the connection. If a student has difficulty accessing numerical meanings from symbols (issues with number sense- eg. has difficulty understanding that a 6 by 1 rectangle would have the same area as a 1 by 6 rectangle, the teacher will encourage the students to use a variety of tools (eg. colored tiles, graph paper, dot paper). If the student misapplies rules or overgeneralizes (eg. length times width is a way to find the area of all polygons), the teacher will encourage students to provide examples, as well as counterexamples to show how and when “rules” should be used and when they should not. The teacher will encourage students to tie all rules to conceptual understanding by asking them why procedures work and/or encouraging them to connect their representations (eg. asking how the counting of squares relate to the multiplication of length times width). For ELL students²: The teacher will simplify sentence structure and limit the use of nonessential or confusing vocabulary when asking questions or giving directions. After the students describe area and perimeter in the context of these problems, the teacher may need to give them these words and may display them on the word wall.		
*****Analysis of Student Learning*****		

Rubric

4	Full Accomplishment	Strategy and execution meet the content, processes and qualitative demands of the task. Communication is judged by effectiveness not length. May have minor errors. Students understand that area refers to the square units inside the rectangle and that the perimeter represents the distance around (and in this case, is always 12 feet and represents the fencing around the rectangle). The student explains or shows that they can find the area by counting squares and may also show that they can use repeated addition and/or multiply length times width to calculate the area. The student explains or shows that they can find the perimeter of each rectangle (which is 12 ft in this case) by either counting the length of all sides (adding up the 4 sides), adding the length plus the width and then multiplying by 2 or multiplying the length times 2 and width times 2 and then finding the sum of the products. The student compares the area of several rectangles with a perimeter of 12 to determine the rectangle with the greatest area as the one with the most space. Minor error: The student may not recognize that the 3ft by 3ft rectangle is the rectangle with the most area because he/she may not realize that a square is a type of rectangle. (Classification of quadrilaterals is not being assessed.) Minor error: The student may make a careless mistake when adding or multiplying.
3	Proficient: Substantial Accomplishment	Students could work to full accomplishment with minimal feedback, however, the student may not be sure about one or more critical features of the task. Errors are minor. Therefore, teacher is confident that understanding is adequate (but not at the level above) to accomplish the objective. For example, the student may multiply length times width to find the area but not know why that procedure works. The student shows or explains that area is the space inside but doesn't explain about or show the squares inside or have a method for finding the area. The student explains or shows that the perimeter is the distance around the rectangle but doesn't relate it to a method for counting the sides.
2	Marginal: Partial Accomplishment	Part of the task is accomplished but there is a lack of evidence of understanding of the concepts involved. Direct input or further teaching is still required. For example, the student may confuse area with perimeter and show all rectangles created with 12 squares inside (with a constant area and changing perimeter). The student uses the 12 feet to represent the squares inside and doesn't count the length of the sides at all.
1	Unsatisfactory: Little Accomplishment	The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little or no success. For example, the student may draw a few rectangles without labeling the dimensions or draw one rectangle with sides that do not represent the area or perimeter.

*Rubric adapted from Van de Walle, et al (2013). *Elementary and middle school mathematics: Teaching developmentally (8th Edition)*.