Solution for Problems 31-60

31. Answer: $\frac{11\pi}{6}$ The radian measure for 330^o is gotten by multiplying by $\frac{\pi}{180}$.

$$330\left(\frac{\pi}{180}\right)=\frac{11\pi}{6}$$ which we get by dividing 330 and 180 by 30.

32. Answer: $\sin\pi = 0$ $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$, $\sin\frac{\pi}{6}=\frac{1}{2}$ and $\sin\pi = 0$.

So the smallest number is $\sin\pi = 0$.

33. Answer: *7.8* Since $\sin B=\frac{opposite}{hypotenuse}=\frac{5}{AB}=0.64$, we get $AB=\frac{5}{0.64}=7.8$ to the nearest tenth.

34. Answer: $-\frac{2}{\sqrt{3}}$ *or* $-\frac{2\sqrt{3}}{3}$ $\csc\left(\frac{4\pi}{3}\right)=\frac{1}{\sin\left(\frac{4\pi}{3}\right)}$.

Since $\frac{4\pi}{3}$ is in quadrant III, the sign of $\sin\frac{4\pi}{3}$ is negative and equal to $-\frac{\sqrt{3}}{2}$.

So $\csc\left(\frac{4\pi}{3}\right)=\frac{1}{\sin\left(\frac{4\pi}{3}\right)}=-\left(\frac{1}{\frac{\sqrt{3}}{2}}\right)=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$.

35. Answer: $\sin\theta$.

The trigonometric identity for the expansion of
$\sin(a-b)=\sin a\cos b-\sin b\cos a$

So that $\sin(180^o-\theta)=\sin180^o\cos\theta - \sin\theta\cos180^o$.

Using $\sin(180^o)=0$ and $\cos(180^o)=-1$, we simplify

$\sin(180^o-\theta)=0(\cos\theta)-\sin\theta(-1)=\sin\theta$

36. Answer: 1

The Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$ applies for any angle.

37. Answer: $\tan B = \dfrac{x}{\sqrt{4 - x^2}}$

The hypotenuse AB is given as 2, the side AC is given as x so the third side BC is

$\sqrt{4 - x^2}$ by using the Pythagorean Theorem.

$x^2 + (BC)^2 = 2^2$, so that $(BC)^2 = 4 - x^2$, $BC = \sqrt{4 - x^2}$

Now the $\tan B = \dfrac{opposite}{adjacent} = \dfrac{AC}{BC} = \dfrac{x}{\sqrt{4 - x^2}}$.

38. Answer: $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$

Since $\sin(a + b) = \sin a\cos b + \cos a\sin b$ and $\cos(a + b) = \cos a\cos b - \sin a\sin b$,

by letting $a = b = \theta$, we get

$\sin(\theta + \theta) = \sin 2\theta = \sin\theta\cos\theta + \cos\theta\sin\theta = 2\sin\theta\cos\theta$.

$\cos(\theta + \theta) = \cos 2\theta = \cos\theta(\cos\theta) - \sin\theta(\sin\theta) = \cos^2\theta - \sin^2\theta$.

39. Answer: $\theta = 30^o$, 90^o, 150^o.

Factor: $2\sin^2\theta + \sin\theta - 1 = 0$ to get $(2\sin\theta - 1)(\sin\theta + 1) = 0$

$2\sin\theta - 1 = 0$ and $\sin\theta + 1 = 0$

$\sin\theta = \dfrac{1}{2}$ and $\sin\theta = -1$

For $\sin\theta = \dfrac{1}{2}$, there are two solutions in the interval $0 \le \theta < 2\pi$, one in quadrant I and one in quadrant II.

$\theta = 30^o$ and $\theta = 150^o$

For $\sin\theta = -1$, there is one solution, $\theta = 270^o$.

40. Answer: $\theta = 0, \ \frac{\pi}{2}, \ \pi, \ \frac{3\pi}{2}$.

For $\cos 4\theta = 1$, $4\theta = 0 + 2n\pi$ for integers n or $\theta = \frac{2n\pi}{4} = \frac{n\pi}{2}$.

Letting n range from 0,1,2,3 we get four answers in the required domain of $0 \le \theta < 2\pi$.

When $n = 4$, $\frac{4\pi}{2} = 2\pi > 2\pi$.

41. Answer: $\frac{2\pi}{3}$ For $y = \mathrm{A}\sin(\mathrm{B}x + \mathrm{C})$ the period is $\frac{2\pi}{B}$.

Thus the period is $\frac{2\pi}{3}$

42. Answer: $arc\cos\mathrm{A} = \frac{11}{16}$.

Using the Law of Cosines with AC = b = 4, BC = a =6, and AB = c = 8,

$a^2 = b^2 + c^2 - 2bc\cos\mathrm{A}$

becomes $6^2 = 4^2 + 8^2 - 2(4)(8)\cos\mathrm{A}$

$36 = 16 + 64 - 64\cos\mathrm{A}$

$36 - 80 = -64\cos\mathrm{A}$

$-44 = -64\cos\mathrm{A}$

$\cos\mathrm{A} = \frac{11}{16}$

$\mathrm{A} = arc\cos\left(\frac{11}{16}\right)$

43. Answer: π

$arc\sin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$, so that $4arc\sin\left(\frac{1}{\sqrt{2}}\right) = 4\left(\frac{\pi}{4}\right) = \pi$.

The $arc\sin x$ is the angle θ whose sine is x; $\sin\theta = x$ where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

44. Answer : $\cot\theta$

Using the relations that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$, we conclude that

$$\cos^2\theta(\tan\theta)(\csc^2\theta) = \cos^2\theta\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin^2\theta}\right) = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

45. Answer: 4
For $f(x) = -x^2 + 5$,
$f(1) = -1^2 + 5 = -1 + 5 = 4$. Think of $-\mathbf{1}^2$ as $-\mathbf{1}(1)^2$.
Remember the order of operations require that we square **1** first and then multiply by -1.

46. Answer: $\frac{3}{5}$ Solving for y we get $y = \frac{3}{5}x - \frac{1}{5}$, so the slope is the coefficient of x, which is $\frac{3}{5}$.

47. Answer: $y = -\frac{3}{4}x - \frac{7}{4}$
The point slope formula for a line passing through the point (x_o, y_o)

with slope $\boldsymbol{m}$ is $y - y_o = \boldsymbol{m}(x - x_o)$
For our problem $(x_o, y_o) = (3 - 4)$ and $m = -\frac{3}{4}$.
$y - y_o = \boldsymbol{m}(x - x_o)$

Thus substituting for x_o, y_o, m
we get

$$y - (-4) = -\frac{3}{4}(x - 3)$$

$$y + 4 = -\frac{3}{4}x - \frac{3}{4}(-3)$$

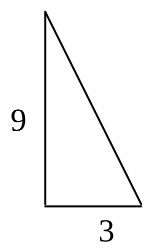
$$y + 4 = -\frac{3}{4}x + \frac{9}{4}$$

$$y = -\frac{3}{4}x + \frac{9}{4} - 4$$

$$y = -\frac{3}{4}x + \frac{9 - 16}{4}$$

$$y = -\frac{3}{4}x - \frac{7}{4}$$

48. Answer: $3\sqrt{10}$ The length of the top (and bottom) is 10 – 7 = 3 and the length of the sides is 7 –(–2) = 7 + 2 = 9. Since the angles of a rectangle are right angles the diagonal, and the adjacent sides form a right triangle with the diagonal as the hypotenuse.



The diagonal, ***d***, and the sides satisfy the Pythagorean Theorem.

$d^2 = 3^2 + 9^2 = 9 + 81 = 90$

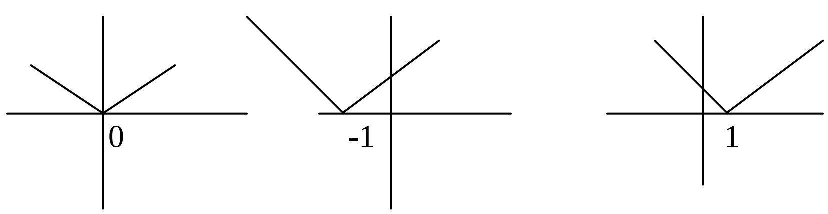
$d = \sqrt{90} = \sqrt{9(10)} = \sqrt{9}\sqrt{10} = 3\sqrt{10}$

49. Answer: $2x + a$.
If $f(x) = x^2$

$$\frac{f(x+a) - f(x)}{a} = \frac{(x+a)^2 - x^2}{a} = \frac{x^2 + 2ax + a^2 - x^2}{a}$$

$$\frac{2ax + a^2}{a} = \frac{a(2x+a)}{a} = 2x + a$$

50. Answer:



$|x|$ is defined as x for $x \geq 0$ which is the right hand line in the first diagram and as $-x$ for $x < 0$ which is the left hand line in the first diagram.

In general the graph of $f(x - c)$ shifts the graph of $f(x)$, c units along the x-axis.

Thus $|x - 1|$ shifts the graph one unit to the left since $x + 1 = x - (-1)$ where $c = -1$.
In a similar way $|x - 1|$ shifts the original graph one unit to the right.

51. Answer: $y = lnx + 2$
For $x = e^{y-2}$, take the natural logarithm of both sides to get
$\ln x = y - 2$,
$y = lnx + 2$.

52. Answer: $x = 8$

The axis of symmetry of a parabola of the form $y = ax^2 + bx + c$, is $x = -\frac{b}{2a}$.

Thus $x = -\frac{(16)}{2(-1)} = 8$

53. Answer: $f(g(x)) = 9x + 1$; $g(f(x)) = \sqrt{9x^2 + 1}$

$f(g(x)) = f(\sqrt{x}) = 9(\sqrt{x})^2 + 1 = 9x + 1$

$f(g(x)) = f(\sqrt{x}) = 9(\sqrt{x})^2 + 1 = 9x + 1$

Also $g(f(x)) = \sqrt{9x^2 + 1}$. Note $\sqrt{9x^2 + 1} \neq 3x + 1$
because the radical does not distribute over addition.

54. Answer: $x = 1$

$\frac{2x - 1}{x^2} = 1$ becomes, after multiplying by x^2,

$2x - 1 = x^2$

Moving all the terms to one side:

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$x = 1$

55. Answer: Domain $x \geq 4$ or $x \leq -4$, Range $y \geq 0$

Since the expression under the radical sign must be non-negative we have

$x^2 - 16 \geq 0$
$x^2 \geq 16$
$\sqrt{x^2} \geq \sqrt{16}$
$|x| \geq 4$

$x \geq 4$ or $x \leq -4$

56. Answer: $x = \frac{1}{2}$, $x = -3$

To find the points of intersection of two curves set the y values equal to each other.

$2x^2 = 3 - 5x$

$2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

$2x - 1 = 0$ $\qquad$ $x + 3 = 0$

$2x = 1$ $\qquad$ $x = -3$

$x = \frac{1}{2}$

57. Answer: -4

Rewrite $\log_2\left(\frac{1}{16}\right) = \log_2\left(\frac{1}{2^4}\right) = \log_2\left(2^{-4}\right) = -4$

58. Answer: $\frac{1}{2}\ln(x^2+1) - \ln x$

The log rules for division and power are $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ and $\ln(a^p) = p\ln a$

we have $\ln\left(\frac{\sqrt{x^2+1}}{x}\right) = \ln\left(\sqrt{x^2+1}\right) - \ln x = \ln\left(x^2+1\right)^{\frac{1}{2}} - \ln x = \frac{1}{2}\ln(x^2+1) - \ln x$.

59. Answer: 3 real roots, 0, -4, 4

$x(x^2 - 16)(x^2 + 16) = 0$

$x = 0,\quad x^2 - 16 = 0,\quad x^2 + 16 = 0$

$(x - 4)(x + 4) = 0\qquad x^2 = -16$

$x = 4,\ x = -4\qquad$ no real solutions

60. Answer: Domain $x > 0$, Range $-\infty < x < \infty$, x – intercept is 1.
As $x \to \infty$, $\ln x \to \infty$, and as $x \to 0^+$, $\ln x \to -\infty$.